CAN MEASUREMENT OF θ_{13} TELL US ABOUT QUARK-LEPTON UNIFICATION ?

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Abstract

We argue that a high precision measurement of the neutrino mixing parameter θ_{13}, within a three neutrino seesaw framework can throw important light on the question of whether the quarks and leptons unify into a single matter at high scales. Based on a number of examples, we conclude that a value of $\theta_{13} \leq \frac{\Delta m^2_{\odot}}{\Delta m^2_A} \simeq 0.04$ would require at the minimum a pure leptonic interchange symmetry between μ and τ generations for its natural theoretical understanding and will disfavor a quark-lepton unification type theory such as those based on $SU(4)_c$ or $SO(10)$ whereas a bigger value would leave open the possibility of quark lepton unification.

I. INTRODUCTION

There are several reasons to think that quarks and lepton may be two different manifestations of the same form of matter. The first hint arises from the observed similarities between weak interaction properties of quarks and leptons. This is generally known as quark-lepton symmetry and even though it manifests itself only in the left-handed helicity sector of quarks and leptons, it is often considered as a hint of further unification among these two very different kinds of matter. The second comes from the attractive hypothesis of grand unification of matter and forces which argues that at very short distances, all forces and all matter unify. Even though there is no direct experimental evidence for grand unification, the apparent couplings unification in a simple supersymmetric model keeps this as a popular idea in the modern particle theory.

There are of course many observations that provide strong distinctions between quarks and leptons: for instance,

- Quarks have strong interactions whereas leptons do not.
- The standard model has both up and down right handed quarks experiencing the gauge forces whereas in the lepton sector only the right handed charged lepton (which is the analog of the down type quark) experiences the gauge force and the right handed neutrino which is the analog of the up quark does not.
- The mixing pattern among quarks of different generations is very different from that among the leptons. For the quarks, we have the "nearest neighbour rule" i.e. $\theta_{12} \simeq 13^o$, $\theta_{23} \simeq 2^o$ and $\theta_{13} \simeq 0.3^o$ whereas for the leptons one has $\theta_{23} \simeq 45^o$; $\theta_{12} \simeq 32^o$ whereas $\theta_{13} \leq 12^o$.

One might therefore conclude that, while it is tempting to speculate about quark lepton unification, the above experimentally observed facts rule out such a possibility. Such conclusion however may be quite premature, since it could very easily be that

quark-lepton unification is not manifest at low energies but could be present at very high energies. In fact there exist very compelling theoretical frameworks that unify quarks and leptons and yet are fully compatible with the above observations. It is then important to look for low energy signals for such unification.

In this talk, I will argue that there may exist such a signal in the neutrino sector. In order to advance my argument, I need to specify the theretical framework within which this conclusion is valid. I will make the minimal set of assumptions, which are quite plausible and are popular in the discussion of neutrino mixings.

(i) There are three generations of neutrinos and they are Majorana fermions.

(ii) The smallness of neutrino masses is explained by the seesaw mechanism[1].

(iii) The high scale quark-lepton unification theory is based on the Pati-Salam $SU(4)_c$ group or SO(10)[2].

(iv) The charged leptons do not contribute significantly to neutrino mixings.

The assumption (ii) would in fact be required in some form if we want to understand why the quark and lepton mixing angles are different in a Q-L unified theory; since the different neutrino mixings could then be attributed to an apppropriate flavor structure among the right handed neutrinos[3]. Within this set of assumptions, three examples are presented where high scale quark-lepton unification implies $\theta_{13} \sim \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}} \simeq 0.1$ or so. I then argue that if $\theta_{13} \leq \frac{\Delta m^2_\odot}{\Delta m^2_A} \simeq 0.04$, then a fine tuning among the different parameters of the theory is required and a natural way to guarantee such small values is to have a purely leptonic symmetry i.e. $\mu - \tau$ interchange, a symmetry not apparently shared by quarks. In such a situation, it is very unlikely that there will be quark lepton unification at high scale.

This talk is organized as follows: in sec. 2, seesaw mechanism is discussed and general arguments are given in favor of the basic thesis of this paper. In sec. 3, three models that embed quark-lepton unifying group $SU(4)_c$ are shown to predict "large" θ_{13} as argued in the introduction. In sec. 4, it shown how $\mu - \tau$ interchange symmetry can lead to "small" values of θ_{13}.

II. SEESAW ENABLES NEUTRINO MIXINGS TO BE LARGE WITHOUT CONFLICTING WITH QUARK-LEPTON UNIFICATION

In this section, we discuss how seesaw mechanism avoids an obvious conflict between quark lepton unification and the vastly different mixing patterns between quarks and leptons. To see why such a conflict would even be contemplated, let us work with the gauge group $SU(2)_L \times SU(2)_R \times SU(4)_c$ subgroup[2] under which the fermions transform as follows:

$$\mathbf{\Psi} \; = \; \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix} \tag{1}$$

This group clearly unifies quarks with leptons. Therefore in the "symmetry limit" one would have $M_u \; = \; M_{\nu^D}$ and $M_\ell \; = \; M_d$. Therefore one might suspect that quark and neutrino mass matrices could be similar. On the hand, it is well known that in a basis where the up quark mass matrix is diagonal, the down quark mass matrix has the generic form:

$$M_d \; \approx \; m_b \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \tag{2}$$

where $\lambda \sim 0.22$ (the Cabibbo angle) and the matrix elements of the above matrix are supposed to represent only the approximate orders of magnitude. On the other hand, for leptons in the basis where charged leptons are mass eigenstates, the neutrino Majorana mass matrix is given by:

$$M_\nu \; = \; \frac{\sqrt{\Delta m^2_A}}{2} \begin{pmatrix} c\epsilon & b\epsilon & d\epsilon \\ b\epsilon & 1 + a\epsilon & -1 \\ d\epsilon & -1 & 1+\epsilon \end{pmatrix} \tag{3}$$

where we have assumed the neutrino mass hierarchy to be normal; $\epsilon \simeq \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}} \approx \lambda$ and parameters a, b, c, d are of order one.

Note however that if small neutrino masses arise from the seesaw mechanism, then their mass matrices arise from either of the following two formulae depending

on whether the theory has asymptotic parity symmetry or not and they are:

$$M_\nu \;=\; -\,M_D^T M_R^{-1} M_D \tag{4}$$

which is the type I seesaw formula where M_R is the right handed neutrino mass matrix. In theories with parity symmetry, one gets instead the type II seesaw formula[4]:

$$M_\nu \;=\; f v_L - h_\nu^T f_R^{-1} h_\nu \left(\frac{v_{wk}^2}{v_R}\right) \tag{5}$$

Therefore even though in the $SU(4)_c$ limit, the quark and lepton mass matrices are identical, one could arrange the structure of M_R such that, the large neutrino mixings arise purely from that structure. For instance, if we take $[M_R]^{-1}_{ij} \;=\; (\mu_{ij})^{-1}$, then the condition for near maximal atmospheric mixing is $\frac{\mu_{22}}{m_c^2} \sim \frac{\mu_{33}}{m_t^2} \sim \frac{\mu_{23}}{m_c m_t}$ and that for solar is $\frac{\mu_{13}}{m_u} \sim \frac{\mu_{33}}{m_t} \cdot \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}}$. Clearly, the above relations imply severe fine tuning among the right handed neutrino masses. In any case, for this toy example we can calculate the neutrino mixings as follows. We now get U_ν of the form:

$$U_\nu \;=\; \begin{pmatrix} c & s & \delta \\ \frac{s}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \tag{6}$$

with a small value for the parameter δ. Since the observed neutrino mixing matrix $U_{PMNS} \;=\; U_\ell^\dagger U_\nu$, we need U_ℓ, which can be obtained in the exact quark-lepton symmetric limit to be $U_\ell \;=\; U^\dagger_{CKM}$. From this, we find that $\theta_{13} \simeq \sum_\alpha [U^\dagger_{CKM}]_{1\alpha}[U_\nu]_{\alpha 3} \sim \delta + \frac{\lambda}{\sqrt{2}} + \frac{\lambda^3 a}{\sqrt{2}}$ (where a is of order one). This leads to $\theta_{13} \simeq 0.15$ which by our definition is "large" without any fine tuning of parameters. While this is a toy model, we see below that all the examples with quark-lepton symmetry that we have explored, something similar happens leading to the generic prediction that θ_{13} is "large".

III. SIMPLE BREAKINGS OF QUARK-LEPTON UNIFICATION AND "LARGE" θ_{13}

In the above example, the quark-lepton symmetry is assumed to be exact for the Yukawa couplings that lead to charged fermion masses. In this section, we depart

from this simple example and consider models with simple breaking of $SU(4)_c$ in the Yukawa couplings that lead to charged fermion masses and argue that in this case we also get θ_{13} to be large.

A. An $SU(2)_L \times SU(2)_R \times SU(4)_c$ example

Let us consider a model with three sets of Higgs fields: $\phi(2,2,0)$, $\Sigma(2,2,15)$ and $\Delta(3,1,10) \oplus \Delta^c(1,3,\bar{1}0)$. The Yukawa superpotential of this model is:

$$W \;=\; h\Psi\phi\Psi^c \;+\; f\Psi\Sigma\Psi^c \;+\; f_\nu(\Psi\Psi\Delta + \Psi^c\Psi^c\Delta^c) \tag{7}$$

After spontaneous breakdown of the gauge symmetry, we get for the fermion mass matrices

$$\begin{aligned} M_u &= h\kappa_u + f v_u \\ M_d &= h\kappa_d + f v_d \\ M_\ell &= h\kappa_d - 3 f v_d \\ M_{\nu_D} &= h\kappa_u - 3 f v_u \end{aligned} \tag{8}$$

where $\kappa_{u,d}$ are the vev's of the up and down standard model type Higgs fields in the $\phi(2,2,0)$ multiplet and $v_{u,d}$ are the corresponding vevs for the same doublets in $\Sigma(2,2,15)$. Note that there are 13 parameters in the above equations and there are 13 inputs (six quark masses, three lepton masses and three quark mixing angles and weak scale). Thus all parameters of the model that go into fermion masses are determined.

To determine the light neutrino masses, we use the seesaw formula in Eq. (5), i.e.

$$M_\nu \;=\; f_\nu v_L - h_\nu^T f_\nu^{-1} h_\nu \left(\frac{v_{wk}^2}{v_R}\right). \tag{9}$$

Now we note from Eq.(8), that there is a sum rule relating the charged lepton and quark mass matrices i.e.

$$k\tilde{M}_\ell = \tilde{M}_u + r\tilde{M}_d \tag{10}$$

where $m_3\tilde{M} = M$ (m_3 being the mass of the third generation fermion. Fitting the τ and μ masses implies that k, r are of order one. Therefore, the form of the charged lepton mass matrix is roughly of the same form as the quark mass matrices. Therefore the argument of the previous section (i.e. exact $SU(4)_c$ case applies here too and leads to a "large" θ_{13}.

This model grand unifies to a class of minimal R-parity conserving SO(10) model[5] discussed recently[6, 7, 8, 9, 10, 11]. The $\phi(2,2,1)$ and $\Sigma(2,2,15)$ multiplets are embedded into the **10** and **126** dimensional representations of SO(10). An interesting advantage of the SO(10) unification noted in [6] is that the triplets $\Delta \oplus \Delta^c$ along with Σ become part of the **126** Higgs representation. As a result, we get $f = f_\nu$ making the model quite predictive in the neutrino sector. The model leaves R-parity as an automatic symmetry of the low energy Lagrangian leading to a naturally stable dark matter in this case.

The model also leads naturally to large neutrino mixing angles without the need for any fine tuning of right handed neutrino masses as in the model discussed in the previous section. To see this happens, note that as already noted earlier, any theory with asymptotic parity symmetry leads to type II seesaw formula. When the triplet term in the type II seesaw dominates the neutrino mass, we have the neutrino mass matrix $M_\nu \propto f$, where f matrix is the **126** coupling to fermions discussed earlier. Using the above equations, one can derive the following sumrule (sumrule was already noted in the third reference of Ref.[7].).

$$M_\nu \; = \; c(M_d - M_\ell) \tag{11}$$

where numerically $c \approx 10^{-9}$ GeV. To see how this leads to large atmospheric and solar mixing, let us work in the basis where the down quark mass matrix is diagonal. All the quark mixing effects are then in the up quark mass matrix i.e. $M_u = U^T_{CKM} M^d_u U_{CKM}$. As already noted the minimality of the Higgs content leads to the following sumrule among the mass matrices:

$$k\tilde{M}_\ell \; = \; r\tilde{M}_d + \tilde{M}_u \tag{12}$$

where the tilde denotes the fact that we have made the mass matrices dimensionless by dividing them by the heaviest mass of the species i.e. up quark mass matrix by m_t, down quark mass matrix by m_b etc. k, r are functions of the symmetry breaking parameters of the model. Using the Wolfenstein parameterization for quark mixings, we can conclude that that we have

$$M_{d,\ell} \;\approx\; m_{b,\tau}\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \tag{13}$$

where $\lambda \sim 0.22$ and the matrix elements are supposed to give only the approximate order of magnitude.

An important consequence of the relation between the charged lepton and the quark mass matrices in Eq. (10) is that the charged lepton contribution to the neutrino mixing matrix i.e. $U_\ell \simeq \mathbf{1} + O(\lambda)$ or close to identity matrix. As a result the neutrino mixing matrix is given by $U_{PMNS} \;=\; U^\dagger_\ell U_\nu \simeq U_\nu$, since in U_ℓ, all mixing angles are small. This satisfies one the criteria listed in the introduction. Thus the dominant contribution to large mixings will come from U_ν, which in turn will be dictated by the sum rule in Eq. (11). Let us now see how how this comes about.

As we extrapolate the quark masses to the GUT scale, due to the fact that $m_b - m_\tau \approx m_\tau\lambda^2$ for a range of values of tanβ, the neutrino mass matrix $M_\nu \;= c(M_d - M_\ell)$ takes roughly the form

$$M_\nu \;= c(M_d - M_\ell) \approx \; m_0\begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix} \tag{14}$$

This mass matrix is in the form discussed in Eq. (3) (when λ is factored out in Eq. (3)). It is then easy to see that both the θ_{12} (solar angle) and θ_{23} (the atmospheric angle) are now large. The detailed magnitudes of these angles of course depend on the details of the quark masses at the GUT scale. Using the extrapolated values of the quark masses and mixing angles to the GUT scale, the predictions of this model for various oscillation parameters are given in Ref.[9]. The predictions for

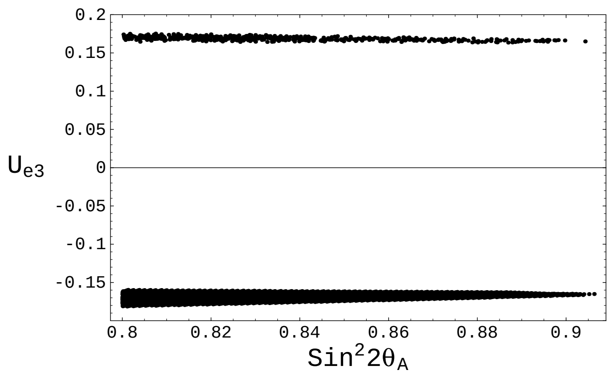

FIG. 1: The figure shows the predictions of the minimal SO(10) model for $sin^2 2\theta_A$ and U_{e3} for the allowed range of parameters in the model. Note that U_{e3} is very close to the upper limit allowed by the existing reactor experiments.

the solar and atmospheric mixing angles fall within 3 σ range of the present central values. Specifically the prediction for U_{e3} (see Fig. 1) can be tested in various reactor experiments[12] and at MINOS as well as other planned Long Base Line neutrino experiments such as Numi-Off-Axis (NoVA), JPARC etc.[13].

There is a simple explanation of why the U_{e3} comes out to be large. This can also be seen from the mass sumrule in Eq.11. Roughly, for a matrix with hierarchical eigen values as is the case here, the mixing angle $tan 2\theta_{13} \sim \frac{M_{\nu,13}}{M_{\nu,33}} \simeq \frac{\lambda^3 m_\tau}{m_b(M_U) - m_\tau(M_U)}$. Since to get large mixings, we need $m_b(M_U) - m_\tau(M_U) \simeq m_\tau \lambda^2$, we see that $U_{e3} \simeq \lambda$ upto a factor of order one. Indeed the detailed calculations lead to 0.16 which is not far from this value.

The model as discussed above does not have CP violation. One way to accomodate CP violation is to include **120** dimensional Higgs multiplet into the theory[10, 14]. By an appropriate choice of CP symmetry, one can choose the **10** and **126** couplings to be real whereas the **120** coupling is imaginary. This not only introduces CP phases into the theory so that one gets CKM CP violation at the weak scale; it also provides a solution to the SUSY CP problem as well as possibly to the strong CP problem. Despite the presence of the extra Higgs multiplet, which brings in three

new parameters, the model is still predictive for θ_{13} and in fact one gets a lower limit for $\theta_{13} \geq 0.08 - 0.1$ (Fig 2 below).

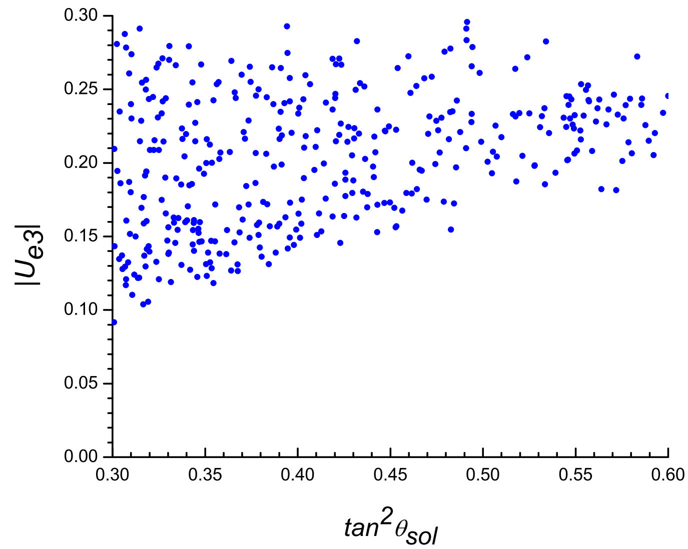

FIG. 2: θ_{13} for the minimal SO(10) model with CKM CP violation. Scatter of points corresponds to different allowed quark mass values. Note that the smallest value is around 0.08.

B. Radiative generation of large mixings with quark-lepton unification

As alluded before, type II seesaw liberates the neutrinos from obeying normal generational hierarchy and instead could easily givbe a quasi-degenerate mass spectrum. This provides a new mechanism for understanding the large mixings. The basic idea is that since at the seesaw scale, one expects quark-lepton unification to be a good symmetry, we expect all mixings angles (i.e. both quark as well as lepton) to be small. Since the observed neutrino mixings are the weak scale observables, one must extrapolate[15] the seesaw scale mass matrices to the weak scale and recalculate the mixing angles. The extrapolation formula is $M_\nu(M_Z) = \mathbf{I} M_\nu(v_R) \mathbf{I}$ where

$\mathbf{I}_{\alpha\alpha} = \left(1 - \frac{h_\alpha^2}{16\pi^2}\right)$. Note that since $h_\alpha = \sqrt{2} m_\alpha / v_{wk}$ (α being the charged lepton index), in the extrapolation only the τ-lepton makes a difference. In the MSSM, this increases the $M_{\tau\tau}$ entry of the neutrino mass matrix and essentially leaves the others unchanged. It was shown[16] that if the muon and the tau neutrinos are nearly degenerate but not degenerate enough in mass at the seesaw scale and have same CP eigenvalue, then the radiative corrections can become large enough so that at the weak scale the two diagonal elements of M_ν become much more degenerate. This leads to an enhancement of the mixing angle to its almost maximal value. This can be seen from the renormalization group equations when they are written in the mass basis[17]. Denoting the mixing angles as θ_{ij} where i, j stand for generations, the equations are:

$$\frac{ds_{23}}{dt} = -F_\tau {c_{23}}^2 \left(-s_{12} U_{\tau 1} D_{31} + c_{12} U_{\tau 2} D_{32}\right), \tag{15}$$

$$\frac{ds_{13}}{dt} = -F_\tau c_{23} {c_{13}}^2 \left(c_{12} U_{\tau 1} D_{31} + s_{12} U_{\tau 2} D_{32}\right), \tag{16}$$

$$\begin{aligned} \frac{ds_{12}}{dt} &= -F_\tau c_{12} \left(c_{23} s_{13} s_{12} U_{\tau 1} D_{31} - c_{23} s_{13} c_{12} U_{\tau 2} D_{32}\right. \\ &\quad \left. + U_{\tau 1} U_{\tau 2} D_{21}\right). \end{aligned} \tag{17}$$

where $D_{ij} = (m_i + m_j)) / (m_i - m_j)$ and $U_{\tau 1,2,3}$ are functions of the neutrino mixings angles. The presence of $(m_i - m_j)$ in the denominator makes it clear that as $m_i \simeq m_j$, that particular coefficient becomes large and as we extrapolate from the GUT scale to the weak scale, small mixing angles at GUT scale become large at the weak scale.

It has been shown recently that indeed such a mechanism for understanding large mixings can work for three generations[18]. The basic idea of Ref.[18] is to identify the neutrino mixing angles with the corresponding quark mixings at the seesaw scale- and assume quasi-degenerate neutrinos. This can be obtailed in models with type II seesaw mechanism and $SU(4)_c$ gauge symmetry. Then by the mechanism of rediative magnification discussed above, the weak scale solar and atmospheric angles get magnified to the desired level while due to the extreme smallness of V_{ub}, the magnified value of U_{e3} remains within its present upper limit.

As noted, such a situation can naturally arise in a parity symmetric model with

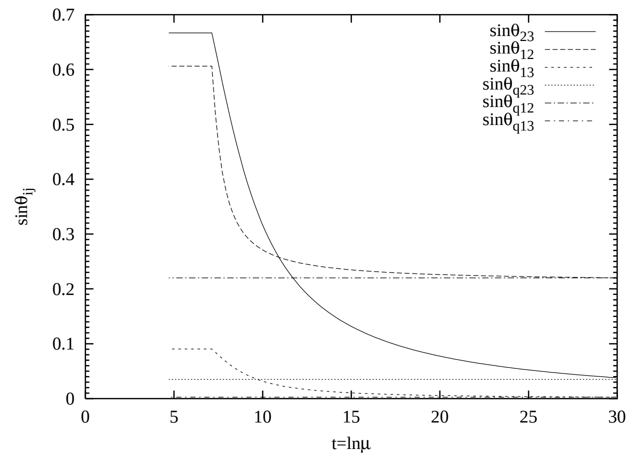

FIG. 3: Radiative magnification of small quark-like neutrino mixings at the see-saw scale to bilarge values at low energies. The solid, dashed and dotted lines represent $\sin\theta_{23}$, $\sin\theta_{13}$, and $\sin\theta_{12}$, respectively.

quark-lepton unification group G_{224} provided one uses both the terms in the type II seesaw formula (Eq. 9) and use a symmetry that leades to $f_\nu = f_0\mathbf{I}$ where $\mathbf{I}$ is the identity matrix. The first term (triplet vev term) then provides the common mass for neutrinos and the standard seesaw term provides the mass splittings. Using the Higgs fields i.e. $\phi(2,2,1)$ and $\Sigma(2,2,15)$, one can get a realistic quark spectrum while keeping the neutrino and quark mixing angles identical at the seesaw scale. They are then extrapolated down to the weak scale using the supersymmetric renormnalization group extrapolation[15]. In figure 3, we show the evolution of the mixing angles to the weak scale. A requirement for this scenario to work is that the common mass of neutrinos must be larger than 0.1 eV, a result that can be tested in neutrinoless double beta experiments.

C. Quark-lepton complementarity and large solar mixing

There has been a recent suggestion[19] that perhaps the large but not maximal solar mixing angle is related to physics of the quark sector. According to this suggestion, the deviation from maximality of the solar mixing may be related to the quark

mixing angle $\theta_C \equiv \theta^q_{12}$ and is based on the observation that the mixing angle responsible for solar neutrino oscillations, $\theta_\odot \equiv \theta^\nu_{12}$ satisfies an interesting complementarity relation with the corresponding angle in the quark sector $\theta_{Cabibbo} \equiv \theta^q_{12}$ i.e.

$$\theta^\nu_{12} + \theta^q_{12} \simeq \pi/4, \tag{18}$$

which seems to be quite well satisfied by present data. While it is quite possible that this relation is purely accidental or due to some other dynamical effects, it is interesting to pursue the possibility that there is a deep meaning behind it and see where it leads. It has been shown in a recent paper that if Nature is quark lepton unified at high scale, then a relation between θ^ν_{12} and θ^q_{12} can be obtained in a natural manner provided the neutrinos obey the inverse hierarchy[20]. This model gives a variation of the quark-lepton complementarity relation (Eq. (18)) and predicts $sin^2\theta_\odot \simeq 0.34$ which agrees with present data at the 2σ level. It also predicts a large $\theta_{13} \sim 0.18$, both of which are predictions that can be tested experimentally in the near future. There are other corrections which affect the value of the solar mixing angle[21] e.g. threshold corrections that can bring the value closer to the present central value. We also note that an operator analysis of QLC has recently been performed in the context of grand unification models with quark-lepton unification[22], where it has been pointed out that obtaining QLC necessarily imples $\theta_{13} \simeq \theta_C$. This prediction is also in line with the general hypothesis of this paper.

IV. $\mu - \tau$ INTERCHANGE SYMMETRY AND SMALL θ_{13}

A question posed by the above discussions is the following: suppose θ_{13} is found to be below the "benchmark" value of 0.04 or so; what does this imply ? It is clear that obtaining such small values in the context of quark lepton unified theories would require fine adjustment among parameters. Typically in physics, when a small parameter requires fine tuning, that is an indication of an underlying symmetry. In the case of θ_{13}, the symmetry turns out to be a simple interchange symmetry between μ and τ generations in the neutrino sector. The motivation to suspect the existence

of such a symmetry is the near maximal value for the atmospheric mixing angle. To see this note that in the neutrino mass matrix given in Eq. (3), if we set $a = 1$ and $b = d$, then the mass matrix is invariant under the interchange of $\mu - \tau$ labels. It is then easy to see by diagonalizing this mass matrix that it leads to $\theta_A = \pi/4$ and $\theta_{13} = 0$. Thus we conclude that a very small value of θ_{13} can be understood if the $\mu - \tau$ symmetry is very nearly exact much the same way one hopes to understand a small value of electron mass as a consequence of chiral symmetry breaking since in the limit of exact chiral symmetry, $m_e = 0$.

In fact if one breaks $\mu - \tau$ symmetry in the $\mu - \tau$ sector by making the neutrino mass matrix to take the following form:

$$M_\nu = \frac{\sqrt{\Delta m^2_A}}{2} \begin{pmatrix} c\epsilon & b\epsilon & b\epsilon \\ b\epsilon & 1+\epsilon & -1 \\ d\epsilon & -1 & 1+\epsilon \end{pmatrix} \tag{19}$$

then it is easy to see that, one gets a nonzero θ_{13} given by:

$$\theta_{13} \simeq \frac{1}{4\sqrt{2}}\epsilon^2 d(1-a) \tag{20}$$

where $\epsilon \simeq \frac{4}{[c+(1+a)/2]+\sqrt{[c-(1+a)/2]^2+8d^2}}\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}}$. Thus the existence of $\mu - \tau$ symmetry allows us to theoretically understand a truly small θ_{13}. An important characteristic of this model is that there is a strong correlation between the value of θ_{13} and deviation of the atmospheric mixing angle from its maximal value. In the last figure of this article[27], we display the correlation as a scatter plot.

In conclusion, I have argued that if the upper limit on the neutrino mixing parameter θ_{13} goes below around 0.04 or so, then this would strongly suggest that there is no quark lepton unification such as those based on the groups $SU(2)_L \times SU(2)_R \times SU(4)_c$ or SO(10) at high scale. This would be a significant step in our search for the ultimate unified theory of forces and matter and therefore provides very strong motivation for high precision experimental search for θ_{13}. Two exceptions to this conclusion are that: (i) there exist sterile neutrinos and/or (ii) the lepton mixings receive dominant contributions from the charged lepton sector[28].

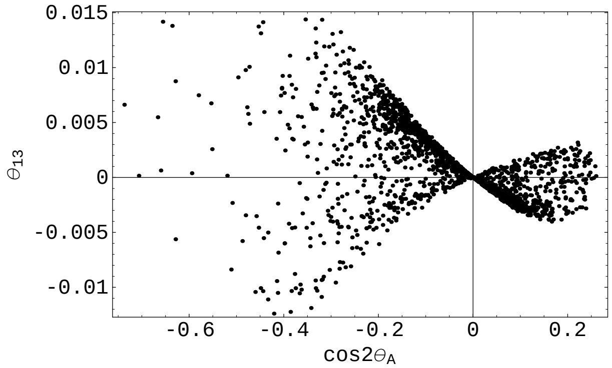

FIG. 4: Departure from $\mu - \tau$ symmetry and correlation between θ_{13} and θ_A.

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